Comparing Rational Functions and Simplified Functions

Learning Objective:

In this lesson, students will simplify rational functions, identify the domain, and determine points of discontinuity.

Standards:

> **Algebra II 7.0** Students evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions.
>
> **Mathematical Analysis 6.0** Students can graph a rational function.
>
> **Calculus 2.0** Students demonstrate knowledge of the graphical interpretation of continuity of a function.

Lesson:

Through T-charts and graphing students will compare the domain of rational functions and simplified functions. Students will determine points of discontinuity in rational functions.

1. Guided practice - Teacher models worksheets #1, #2, and #3 while students fill in their copy of the worksheet.
    - Simplify the rational function
    - Fill in the $f(x)$ and $g(x)$ values in the T-charts
    - Graph both functions and draw a circle where a point is not defined
    - State the domain of each function. If there is a value of x where the function is undefined, identify as a point of discontinuity and state that the simplified function is continuous.

2. You try – Students work in pairs and complete worksheet #4.

3. Suggestions for choral response:
    - The hole in the graph of the rational function is called a__________. [point of discontinuity]
    - The simplified function is a polynomial and polynomials are ___________. [continuous]
    - The graph of the simplified function is continuous everywhere and does not have a _________. [hole]
    - To determine the domain in a rational function, we **must** use the _______________. [original function]

Warm-Up

CST/CAHSEE: Algebra I 12.0

Simplify $\dfrac{6x^2 + 21x + 9}{4x^2 - 1}$ to lowest terms.

A. $\dfrac{3(x+1)}{2x-1}$

B. $\dfrac{3(x+3)}{2x-1}$

C. $\dfrac{3(2x+3)}{4(x-1)}$

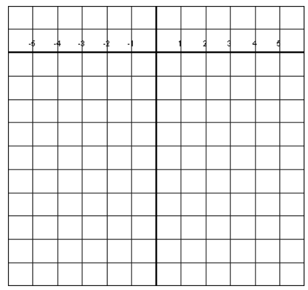
D. $\dfrac{3(x+3)}{2x+1}$

Review: Algebra II 8.0

Given $y = x^2 + 2x - 8$

Find the x intercepts, y intercept and the vertex.

Graph the equation and state the domain.



Current: Algebra II 7.0

Simplify each function and state the value(s) of x that make the function undefined.

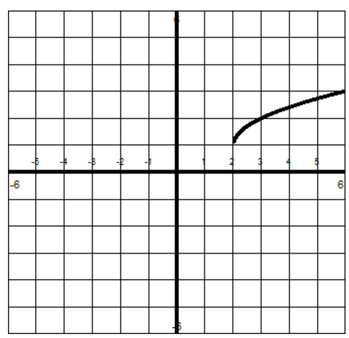
(a) $f(x) = \dfrac{3-x}{x^2 - 3x}$

(b) $f(x) = \dfrac{x^2 - 2x - 15}{x - 5}$

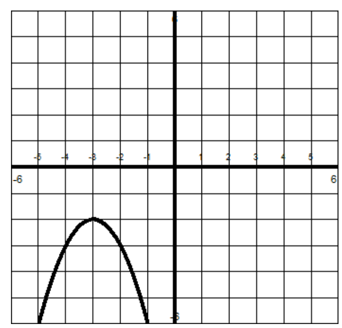
Other: Algebra I 17.0

For each graph shown, state the domain.

(a)



(b)



Today's Objective/Standards:

Algebra II 7.0 Students evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions.

Mathematical Analysis 6.0 Students can graph a rational function.

Calculus 2.0 Students demonstrate knowledge of the graphical interpretation of continuity of a function.

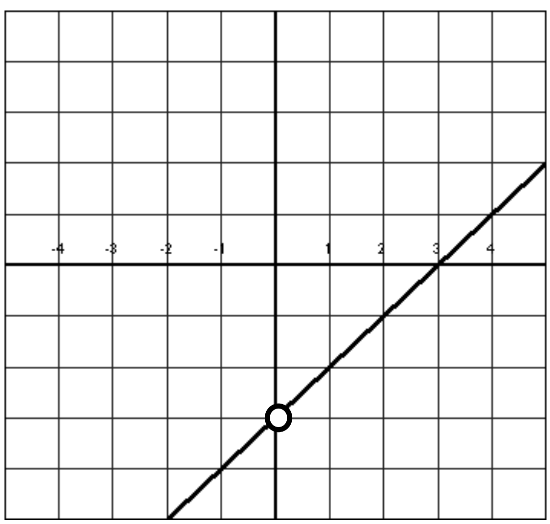
Guided Practice - Worksheet #1 with solutions

Rational Function

$$f(x)=\frac{x^2-3x}{x}$$

The denominator is a factor of the numerator.

x	$f(x)$
-2	-5
-1	-4
0	$\frac{0}{0}$ indeterminate
1	-2
2	-1



Domain: *[All real numbers, $x \neq 0$]*

For what value of *x* is $f(x)=\frac{0}{0}$**?** *[$x=0$]*

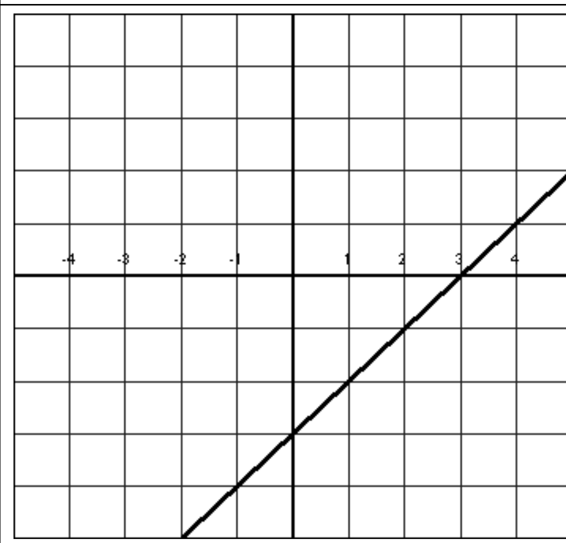
Conclusion: *[This rational function has a point of discontinuity at* $x=0$ **.***]*

Simplified Function

$$g(x)=x-3$$

The function simplifies to a polynomial.

x	$g(x)$
-2	-5
-1	-4
0	-3
1	-2
2	-1



Domain: *[All real numbers]*

Conclusion: *[A polynomial function is continuous and does not have a point of discontinuity.]*

Guided Practice - Worksheet #2 with solutions

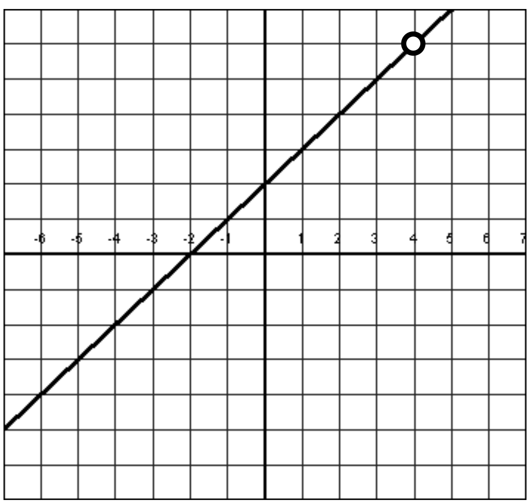
Rational Function	Simplified Function
$f(x) = \dfrac{x^2 - 2x - 8}{x - 4}$ The denominator is a factor of the numerator.	$g(x) = x + 2$ The function simplifies to a polynomial.

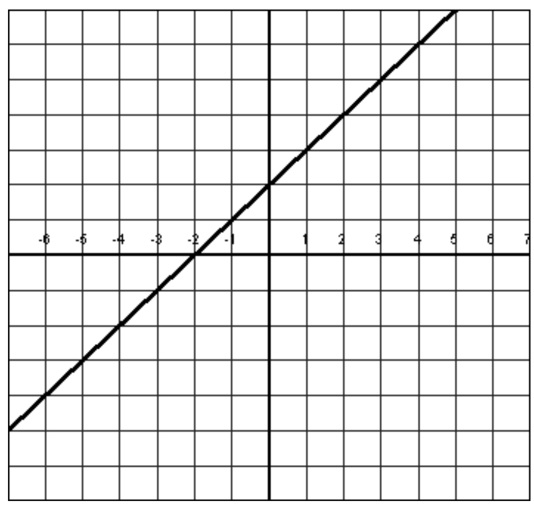
Rational Function

x	$f(x)$
2	4
3	5
4	$\frac{0}{0}$ indeterminate
5	7
6	8

Simplified Function

x	$g(x)$
2	4
3	5
4	6
5	7
6	8





Domain: *[All real numbers, $x \neq 4$]*

What value of *x* is $f(x) = \frac{0}{0}$? *[$x = 4$]*

Conclusion: *[This rational function has a point of discontinuity at $x = 4$.]*

Domain: *[All real numbers]*

Conclusion: *[A polynomial function is continuous and does not have a point of discontinuity.]*

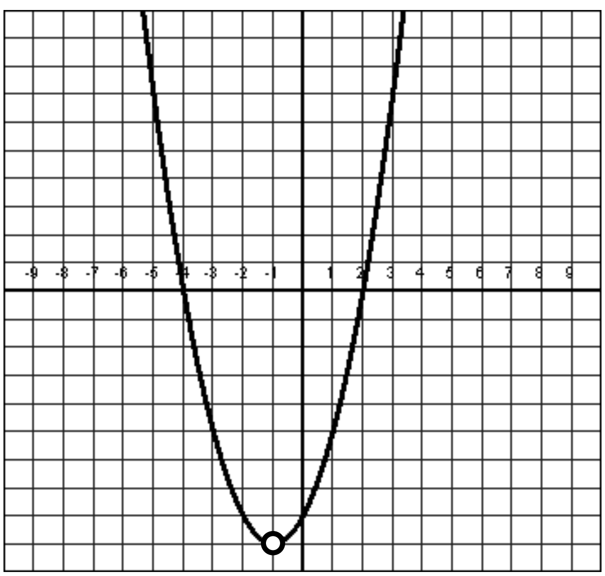
Guided Practice - Worksheet #3 with solutions

Rational Function

$$f(x) = \frac{x^3 + 3x^2 - 6x - 8}{x+1}$$

The denominator is a factor of the <u>numerator</u>.

x	$f(x)$
-4	0
-3	-5
-2	-8
-1	$\frac{0}{0}$ indeterminate
0	-8
1	-5
2	0



Domain: *[All real numbers, $x \neq -1$]*

For what value of *x* is $f(x) = \frac{0}{0}$**?** *[$x = -1$]*

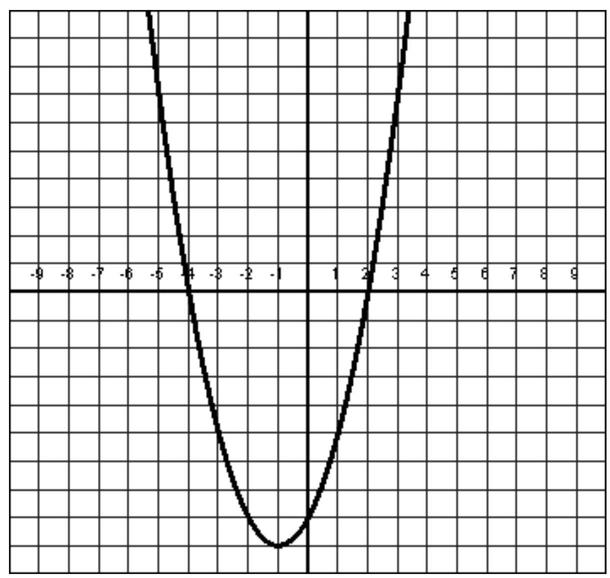
Conclusion: *[This rational function has a point of discontinuity at $x = -1$.]*

Simplified Function

$$g(x) = x^2 + 2x - 8$$

The function simplifies to a <u>polynomial</u>.

x	$g(x)$
-4	0
-3	-5
-2	-8
-1	-9
0	-8
1	-5
2	0



Domain: *[All real numbers]*

Conclusion: *[A polynomial function is continuous and does not have a point of discontinuity.]*

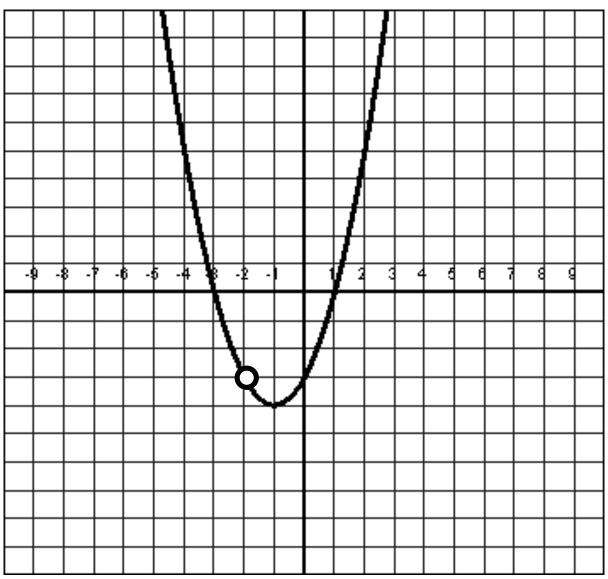
You Try - Worksheet #4 with solutions

Rational Function

$$f(x)=\frac{x^3+4x^2+x-6}{x+2}$$

The denominator is a factor of the numerator.

x	$f(x)$
-5	12
-4	5
-3	0
-2	$\frac{0}{0}$ indeterminate
-1	-4
0	-3
1	0



Domain: *[All real numbers, $x \neq -2$]*

For what value of *x* is $f(x)=\frac{0}{0}$ **?** *[$x=-2$]*

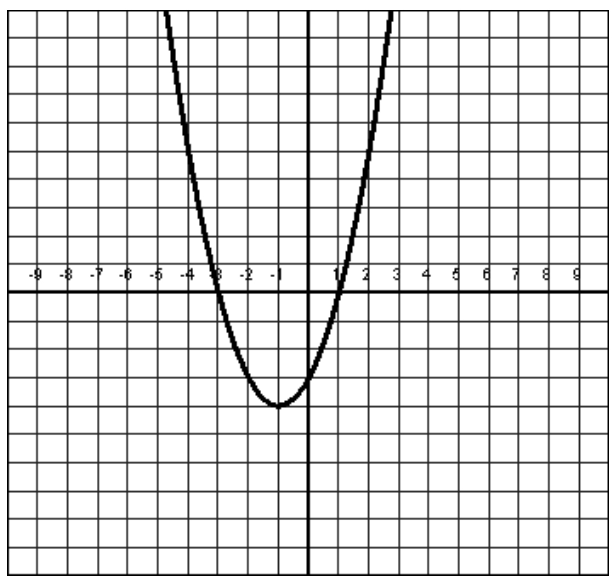
Conclusion: *[This rational function has a point of discontinuity at $x=-2$.]*

Simplified Function

$$g(x)=x^2+2x-3$$

The function simplifies to a. polynomial.

x	$g(x)$
-5	12
-4	5
-3	0
-2	-3
-1	-4
0	-3
1	0



Domain: *[All real numbers]*

Conclusion: *[A polynomial function is continuous and does not have a point of discontinuity.]*

Summary Activities with solutions

1. Have students work in pairs to complete the following table.

	Rational Function	What value of x is excluded from the domain?	Point(s) of discontinuity	What form is the graph of the simplified function? (i.e. linear, quadratic)
a.	$f(x)=\dfrac{-2x^3+9x^2-10x+3}{x-3}$	$x=3$	at $x=3$	quadratic
b.	$f(x)=\dfrac{2x^2+x-1}{2x-1}$	$x=\dfrac{1}{2}$	at $x=\dfrac{1}{2}$	linear
c.	$f(x)=\dfrac{x^3-13x-12}{x^2-3x-4}$	$x=-1$, $x=4$	at $x=-1$ and at $x=4$	linear

2. What are the similarities and differences between the graphs of the rational functions and their simplified functions?

- Similarity – same shape
- Difference – point(s) of discontinuity in rational function or hole(s) in graph
- The rational function $f(x)$ in the worksheets agrees with the simplified function $g(x)$ at all points except at points of discontinuity.

3. Write a rational function where the graph of the simplified function is quadratic with a hole at $x=-5$. Verify that when $x=-5$, $f(x)=\dfrac{0}{0}$.

Answers will vary.

Example: $f(x)=\dfrac{x^3+8x^2+17x+10}{x+5}$

$$\begin{aligned} f(-5) &= \frac{(-5)^3+8(-5)^2+17(-5)+10}{-5+5} \\ &= \frac{-125+200-85+10}{0} \\ &= \frac{0}{0} \end{aligned}$$

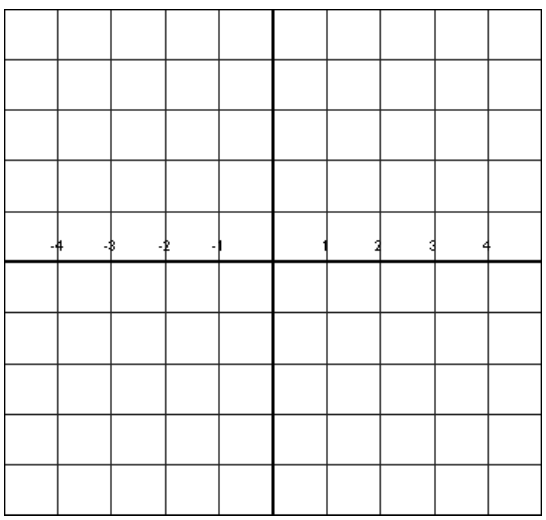
Student Worksheet #1

Rational Function

$$f(x) = \frac{x^2 - 3x}{x}$$

The denominator is a factor of the __________.

x	$f(x)$
-2	
-1	
0	
1	
2	



Domain:

For what value of *x* is $f(x) = \frac{0}{0}$ **?**

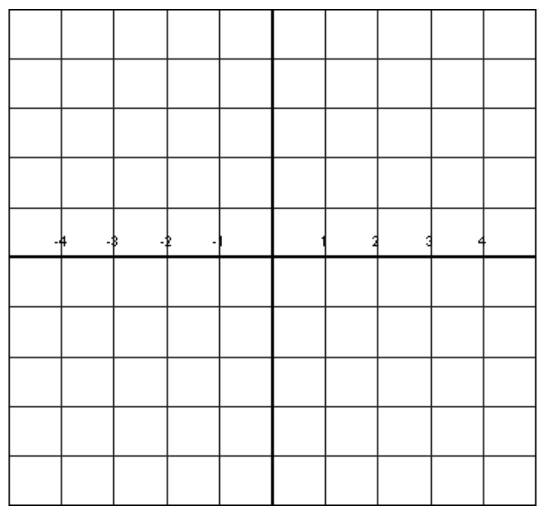
Conclusion:

Simplified Function

$g(x) =$

The function simplifies to a ________________.

x	$g(x)$
-2	
-1	
0	
1	
2	



Domain:

Conclusion:

Student Worksheet #2

<table>
<tr><th>Rational Function</th><th>Simplified Function</th></tr>
<tr><td>

$$f(x) = \frac{x^2 - 2x - 8}{x - 4}$$

The denominator is a factor of the __________.

</td><td>

$$g(x) =$$

The function simplifies to a ______________.

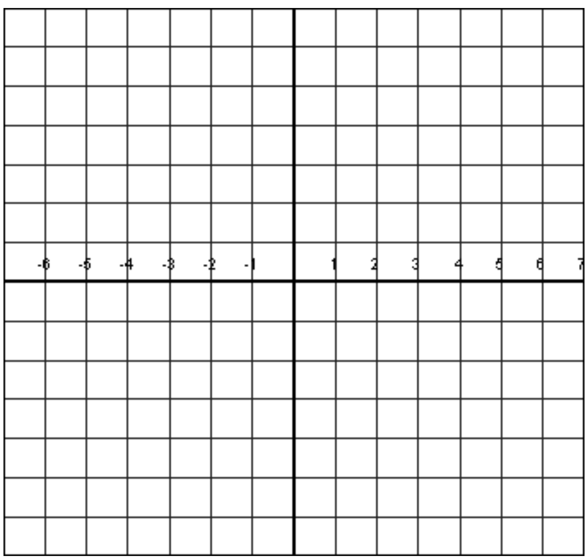
</td></tr>
<tr><td>

x	$f(x)$
2	
3	
4	
5	
6	

</td><td>

x	$g(x)$
2	
3	
4	
5	
6	

</td></tr>
<tr><td>



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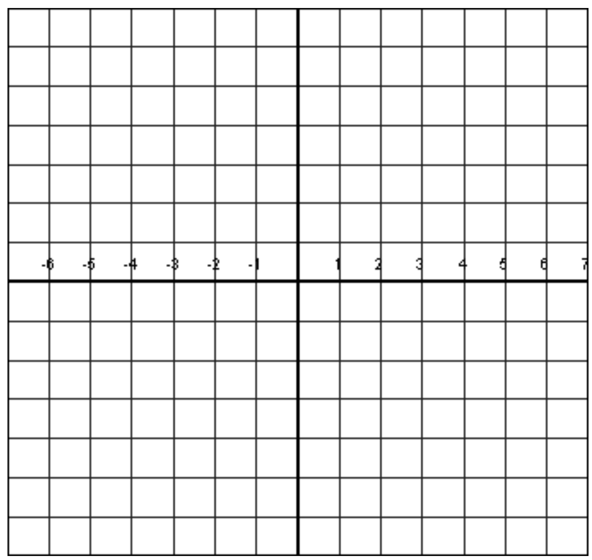


</td></tr>
<tr><td>

Domain:

For what value of *x* is $f(x) = \frac{0}{0}$ **?**

Conclusion:

</td><td>

Domain:

Conclusion:

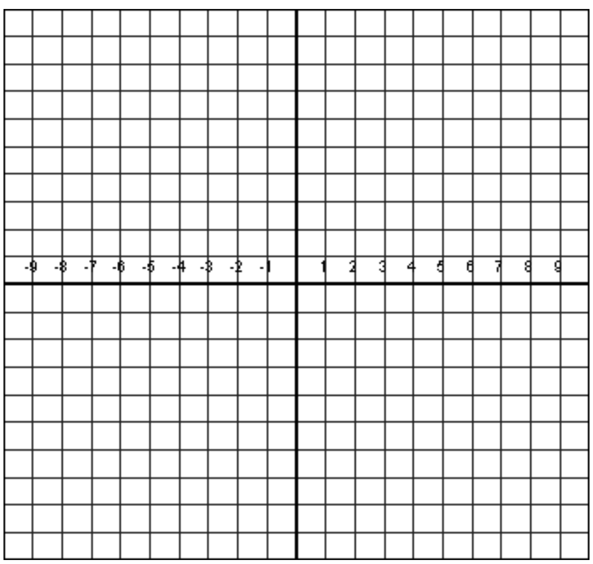
</td></tr>
</table>

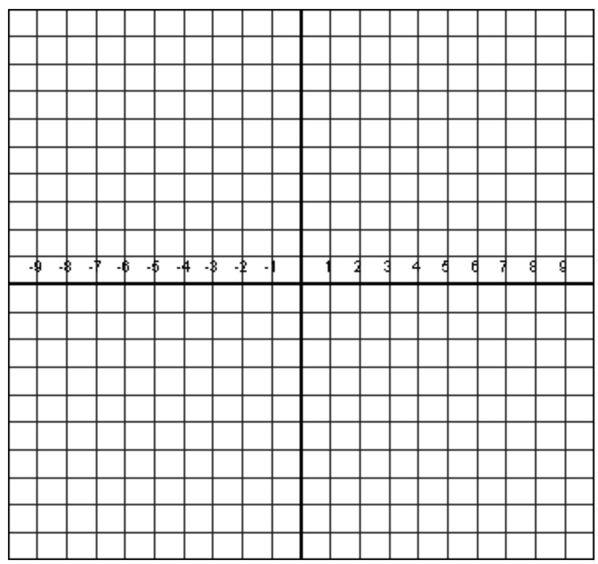
Student Worksheet #3

Rational Function	Simplified Function
$f(x)=\dfrac{x^3+3x^2-6x-8}{x+1}$ The denominator is a factor of the __________.	$g(x)=$ The function simplifies to a _______________.

x	$f(x)$
-4	
-3	
-2	
-1	
0	
1	
2	

x	$g(x)$
-4	
-3	
-2	
-1	
0	
1	
2	





Domain:

For what value of *x* is $f(x)=\dfrac{0}{0}$ **?**

Conclusion:

Domain:

Conclusion:

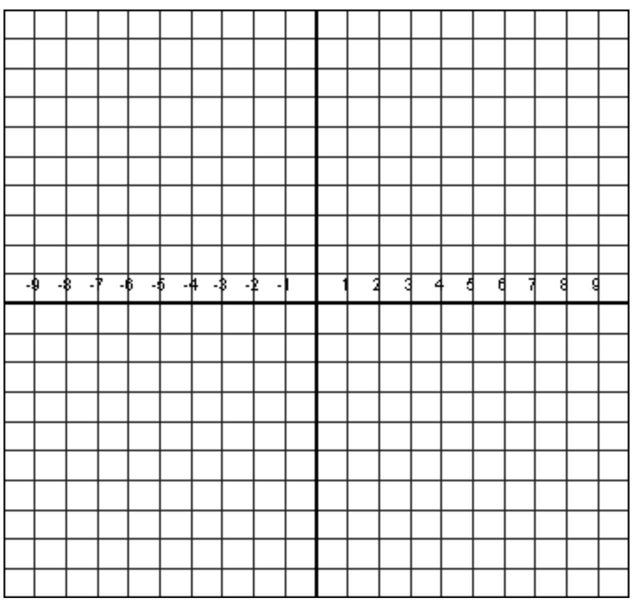
Student Worksheet #4

Rational Function

$$f(x)=\frac{x^3+4x^2+x-6}{x+2}$$

The denominator is a factor of the ___________.

x	$f(x)$
-5	
-4	
-3	
-2	
-1	
0	
1	



Domain:

For what value of *x* is $f(x)=\frac{0}{0}$ **?**

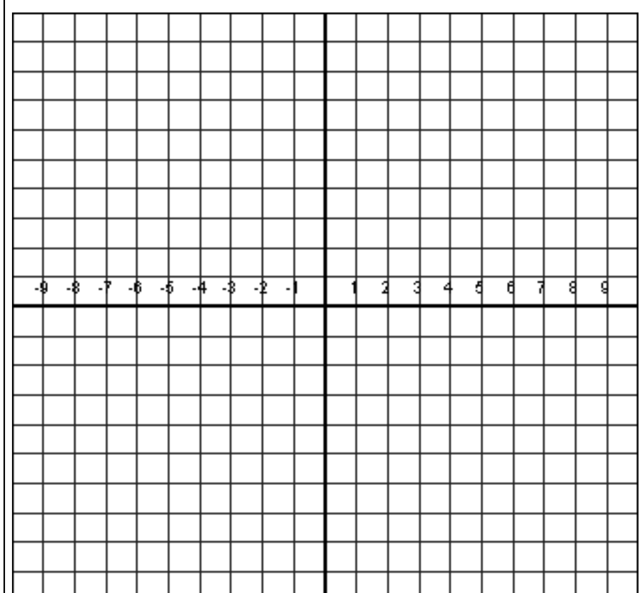
Conclusion:

Simplified Function

$g(x)=$

The function simplifies to a _______________.

x	$g(x)$
-5	
-4	
-3	
-2	
-1	
0	
1	



Domain:

Conclusion:

Summary Activities

1. Have students work in pairs to complete the following table.

	Rational Function	What value of x is excluded from the domain?	Point(s) of discontinuity	What form is the graph of the simplified function? (i.e. linear, quadratic)
a.	$f(x)=\dfrac{-2x^3+9x^2-10x+3}{x-3}$			
b.	$f(x)=\dfrac{2x^2+x-1}{2x-1}$			
c.	$f(x)=\dfrac{x^3-13x-12}{x^2-3x-4}$			

2. What are the similarities and differences between the graphs of the rational functions and their simplified functions?

3. Write a rational function where the graph of the simplified function is quadratic with a hole at $x=-5$. Verify that when $x=-5$, $f(x)=\dfrac{0}{0}$.

Vocabulary and Assessment

Vocabulary:

Rational Expression: A rational expression is an expression in the form $\frac{polynomial}{polynomial}$. .

Rational Function: A rational function is a function where $f(x) = \frac{polynomial}{polynomial}$.

Domain: The domain is the set of x coordinates.

Continuity (conceptual definition): The graph can be drawn without any breaks. From the tactile perspective, the graph can be drawn without lifting the pencil.

Point of Discontinuity (conceptual definition): Where a value for x is not included in the domain of a function because $f(x) = \frac{0}{0}$, which is indeterminate. From the tactile perspective, draw a circle on the graph to represent a point that is not part of the graph.

Suggested assessment questions: These could be used during the lesson or after as assessment questions. If used during the lesson, elect non-volunteers. Encourage students to answer in complete sentences.

- What is the hole in the graph called? [The hole in the function is called a point of discontinuity.]
- What is the difference between the graph of the rational function and the simplified function? [There is a hole in the graph of the rational function.]
- Which function is continuous? [The simplified function is continuous. The polynomial is continuous.]
- What is the domain of the simplified function? [The domain of the simplified function is all real numbers.]
- Which function has a point of discontinuity, the rational function or the simplified function? [The rational function has a point of discontinuity.]
- Why is there a point of discontinuity? [There is a point of discontinuity because there exists a value of x where $f(x) = \frac{0}{0}$.]
- Why is $f(x) = \frac{0}{0}$ not in the T-chart of the simplified function? [$f(x) = \frac{0}{0}$ is not in the T-chart because the simplified function is a polynomial.]
- Why do we use the original function to find the domain of a rational function? [We need to find the x where $f(x) = \frac{0}{0}$, then we exclude this value of x from the domain.]